Geometric Predicates

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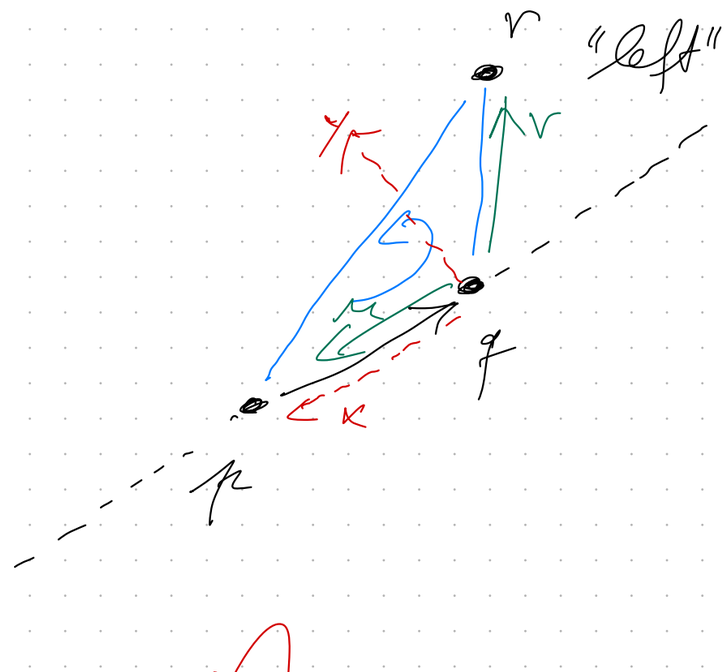
Predicate: "left/right"

Given 3 pts $p, q, r \in \mathbb{R}^2$

Question: is r left of $\overline{pq}$?



Computation:

"left" $\Leftrightarrow$ area $(\triangle pqr) > 0$

$$2 \cdot \text{area}(\triangle pqr) = \begin{vmatrix} p_x & p_y & 1 \\ q_x & q_y & 1 \\ r_x & r_y & 1 \end{vmatrix} = \begin{vmatrix} p_x - q_x & p_y - q_y & 0 \\ q_x & q_y & 1 \\ r_x - q_x & r_y - q_y & 0 \end{vmatrix} =$$

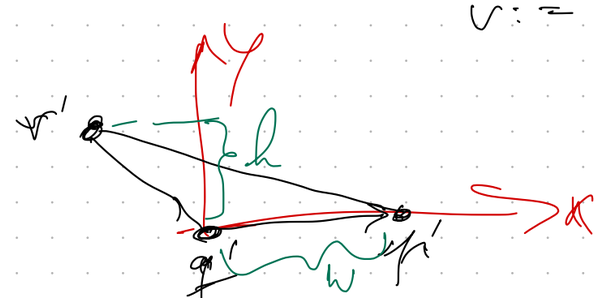
$$= -(p_x - q_x)\cdot(r_y - q_y) + (p_y - q_y)\cdot(r_x - q_x)$$

| def
$u := p - q$
$v := r - q$

$$= -u_x v_y + u_y v_x$$

$$= w \cdot h$$

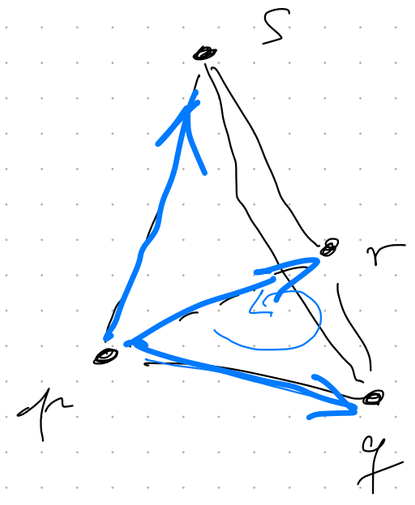
$w = v_y$, $h = u_x$



Predicate: orientation in 3D

Given: 4 pts $p, q, r, s \in \mathbb{R}^3$

Question: is s above/below $\triangle pqr$;
is s in front/back of $\triangle pqr$



Def.:

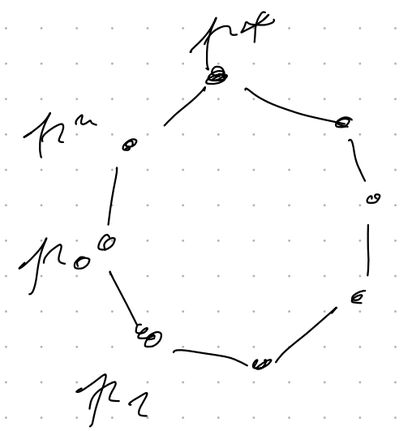
s in front of $p, q, r :\Leftrightarrow$
p, q, r viewed from s appears in CCW order $\Leftrightarrow$
s is on the same side as the normal of $\triangle pqr$ $\Leftrightarrow$
volume $(p, q, r, s) > 0$

$$\text{vol}(pqrs) = \frac{1}{6} \begin{vmatrix} p_x & p_y & p_z & 1 \\ q_x & q_y & q_z & 1 \\ r_x & r_y & r_z & 1 \\ s_x & s_y & s_z & 1 \end{vmatrix}$$

Predicate / Subroutine: Finding extreme pts of polygon

Given: convex pgon $p^0, p^1, \dots, p^n$ (use "wrap-around index arithmetic")

Goal: determine p^* with $p^*_y = \max$

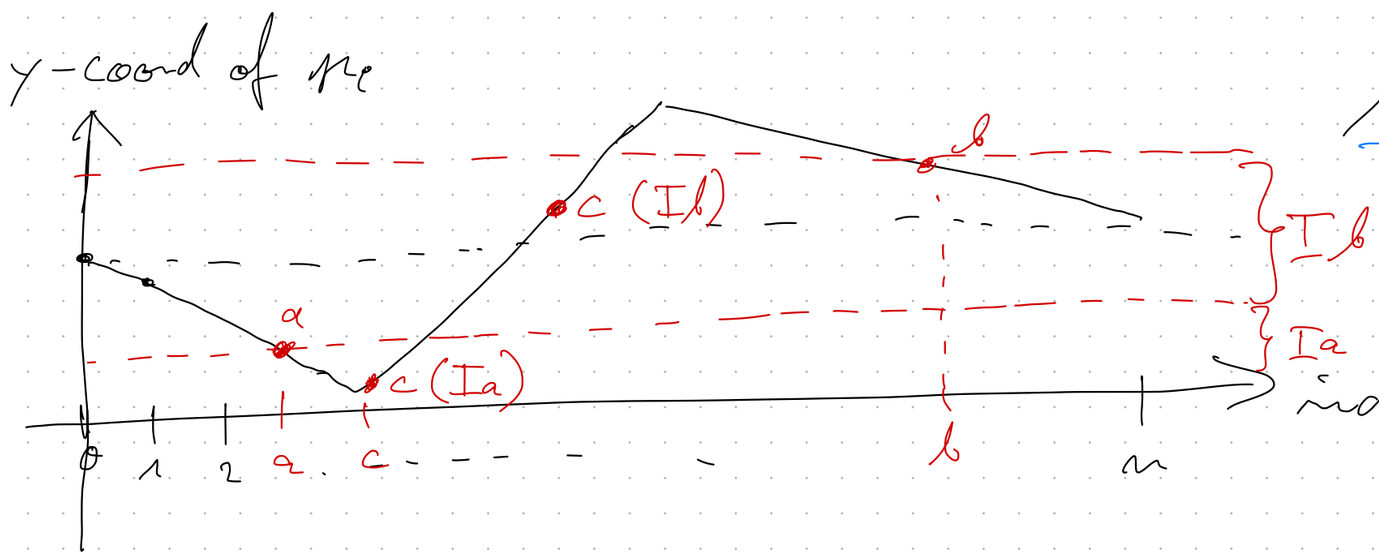


Start with $a = 0$, $b = n$

Assume: p^* is between p_a, p_b

$\Rightarrow c = \frac{1}{2}(a+b)$

Consider following cases:



bitonic sequence

$p^i = (x_i, y_i)$

I a) $y_c < y_a < y_b \Rightarrow$ new bracket $a' := c$, $b' := b$

I b) $y_a < y_c < y_b \Rightarrow$ new bracket $a' = c$, $b' = b$

⋮

$\Rightarrow$ Time $O(\log n)$

Variant: Fibonacci search

Predicate / Subroutine: sort p, q w.r.t polar angle

question: $\varphi_p > \varphi_q$

Solution:

$$\varphi_p > \varphi_q \Leftrightarrow \text{area}(\triangle oqp) > 0$$